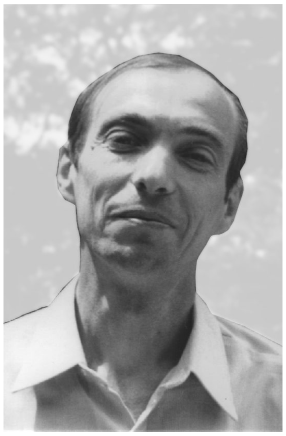
Nikolaí N. Nekhoroshev (Нехорошев)



Nikolaí Nekhoroshev
1946–2008

In 2016 we celebrate the 70-th birthday anniversary of Nikolaí Nekhoroshev, the prominent Russian mathematician, who become widely known for his theory of exponential stability of nearly integrable Hamiltonian systems. The present issue of the journal Regular and Chaotic Dynamics is dedicated to his memory.

Nikolaí Nikolaevich was born on the 2nd of October 1946 in Kursk, a town in central Russia (then USSR), some 500 kilometers south from Moscow. His father (1902–1984) served as a border guard, fought in the war of 1941-45, and after that worked as veterinary paramedic. His mother (1905–1991) was a homemaker, and his three sisters Greta 1929, Eugenia 1934, and Lyudmila 1939, continued to live in Kursk in the 2000s. The older sister Greta recalls that Nikolaí lived in Kursk with his parents, while the sisters had separate homes, and Greta lived in the Far East. As a paramedic, his father also worked on the side, the reward was often a glass of wine ... The father participated little in the family life, and this, apparently, has left its trace. Later, Nikolaí Nikolaevich never tasted any alcohol. The family youngest, the sole and long awaited son, the boy was becoming mother's favorite. She was a high school graduate and took a very active part in his life, went to parent–teacher interviews, and followed his homework. It is likely that such features of his personality as tact, strictness with himself, and meticulousness in work were formed in his young years under her influence. Subsequently, Nikolaí Nikolaevich remained very attached to his mother and mourned painfully her passing away. He never came back to Kursk after her death in 1991.

Very conscientious by nature, Nikolaí excelled in thorough preparations of his homework. His keen interest in mathematics was awakened by his teachers whom he always remembered with gratitude. G. F. Dyominova (Lestchenko), a contemporary of N. N. Nekhoroshev and also a Kursk native, recalls how evening classes in physics and mathematics were organized in the Kursk teacher training college in the early 60s. The professors of the college gave weekly courses on topics which were the most advanced at the time. The participants were learning a great deal. The courses were free and practically all senior grade students of Kursk who liked physics and mathematics were attending them. Usually about 100 listeners assembled in the auditorium and Kolya Nekhoroshev was among them.

At the same time, city and regional Olympiads on physics and mathematics for senior school students were also staged at the Kursk teacher's college. In February 1963, Nikolaí, then a grade ten student of the Kursk polytechnic high school №38 with professional training, participated in the 4th Olympiad in mathematics of Kursk and the region and won the first place. He was sent to the all-Russian mathematics Olympiad for school students as member of a four student team from Kursk. It was during this voyage that Galina Dyominova had met him. She remembers Kolya as a serious and taciturn young man.

We should note that the organization of and preparation for all these events relied largely on the enthusiasm of physics and mathematics teachers who worked long hours with senior students after school, often without any compensation. Their selfless work and devotion paved the way into great mathematics for such gifted pupils as Kolya Nekhoroshev. We like to thank them.

A. N. Kolmogorov and V. I. Arnold in 1964

Upon the results of the all-Russian Olympiad in mathematics, where Nikolaí won a prize, 46 students, among whom he and Galina represented Kursk, were invited to participate in the first mathematics summer school in Krasnovidovo organized by academicians A. N. Kolmogorov and P. S. Aleksandrov. The young doctor of sciences V. I. Arnold, the future scientific adviser of Nikolaí, was one of the lecturers at that school. The 1963 summer school was the beginning of all specialized high schools with extended mathematics curricula in the USSR. Many memories of this remarkable event are still vivid. The lecturers behaved in a very simple and friendly manner. It was inspiring to interact so closely with world class scientists. Galina Dyominova recalls that the students and post-graduates of the mathematical department were both teaching mathematics and counseling. They organized many intellectual games and activities which were enjoyed by most attendants. Nikolaí, however, was not much interested in such divertissements. Instead, he continued to progress in his studies of mathematics. In September of the same year, he won the 3rd prize at the 2nd Olympiad in physics and mathematics of the European part of the USSR and Transcaucasia which was organized by the staff of the Moscow State University (MSU).

First graduation class of the boarding school №18 in 1964, Kolya Nekhoroshev is 9th from the right.

At that time in Moscow, A. N. Kolmogorov organized the boarding school №18 with extended curriculum in physics and mathematics, known today as Specialized research and training A. N. Kolmogorov school associated with MSU[1] where gifted students from the whole country were selected. Admissions to

[1] For more information consult http://internat.msu.ru and http://www.pms.ru

grade 9, 10, and 11 were called for simultaneously. The graduation students of the 11th grade were selected in Krasnovidovo. In order to be admitted, they had to pass examinations in two of the special courses which they attended at the summer school. Upon the results of these tests, 19 students including Nikolaí were selected, see the photograph. So at the age of 17 he left his native Kursk.

V. I. Arnold with the students of the boarding school №18. From left to right: V. I. Arnold, Andrej Kukushkin, Sasha Krygin, Kolya Nekhoroshev (in front), Valera Balin. The daily «Soviet Russia» of March 7, 1964. Photograph by N. S. Safronov.

A. N. Kolmogorov and V. I. Arnold were among the teachers in the boarding school. V. I. Arnold, whom the students called simply Dima, attended particularly to the graduating class. He gave the special course in ordinary differential equations. In the photograph, V. I. Arnold explains to his school students the construction of phase portraits of one degree of freedom systems using the example of motion in a double well potential.

In 1964, now a graduate of the boarding school №18 with extended physics and mathematics curriculum, N. N. Nekhoroshev was admitted to the department of mechanics and mathematics of Moscow State University (MSU), known as «mech-mat». From this moment until the end of his life he stayed attached to this department.

With the student virgin land brigade in Kazakhstan in 1965. Nikolaí is second right, middle row.

Beginning in 1965, Nikolaí was an active participant and organizer of student summer construction brigades with which he traveled several times to Kazakhstan and Sakhalin. He was among the founders and in 1967—the first leader of mekh-mat's most famous student brigade The Tyn Republic (*tyn* is Kazakh for virgin land). Nikolaí Nikolaevich always recalled this time as one of the most remarkable periods of his life.

At the department of mechanics and mathematics, N. N. Nekhoroshev began to participate in the seminar of V. I. Arnold. In 1966, V. I. Arnold—then a young mekh-mat professor aged 29—selected his first group of students that together with N. N. Nekhoroshev included S. Zdravkovska, B. M. Ivlev, A. S. Pyartli, and A. G. Khovanskiı́. The student years where rich in intense interactions with the scientific advisor, in many long walks and discussions about everything in the world. And of course, scientific research was above all. Nikolaı́ Nikolaevich was thinking about a very difficult problem. His work did not go easily. But it was the research work which he initiated in his student years that later brought Nekhoroshev worldwide recognition.

After his graduation from the university in 1969, from October of that year until March 1972, N. N. Nekhoroshev studied in the postgraduate school at the faculty (chair) of differential equations under the supervision of V. I. Arnold. In April 1972, he took the assistant professorship at the same faculty, and in September 1973, he defended his Ph.D thesis. From 1972, Nikolaı́ Nikolaevich together with Yu. S. Ilyashenko and E. M. Landis headed the seminar on ordinary differential equations. In the late 70s, E. M. Landis had distanced himself from the seminar, while the seminars of Nekhoroshev and Ilyashenko separated. After 2001, N. N. Nekhoroshev worked at the chair of calculus.

One of the first scientific papers of N. N. Nekhoroshev was written in 1971 on the basis of his M.Sc. diploma under the supervision of V. I. Arnold at the Moscow State University. It concerned action-angle variables and their generalizations and it turned out to be pioneering in the theory of the so-called non-commutatively integrable systems, i.e., Hamiltonian systems with a complete noncommutative algebra of first integrals. Such a situation occurs, for example, when we consider a system which is invariant under the action of a nonabelian Lie group. N. N. Nekhoroshev formulated conditions under which the joint level sets of first integrals are tori with quasiperiodic dynamics and proved the existence of generalized action-angle variables. In the case of the noncommutative algebra of first integrals, contrary to the case of standard integrability, the dimension of these tori is strictly below one half that of the phase space. Such systems are also sometimes called superintegrable. The ways in which they can be built were subsequently described and numerous examples were given. In each example, the results obtained by Nekhoroshev serve as the principal instrument for the description of dynamics.

The Ph.D thesis of N. N. Nekhoroshev gives the proof of both a remarkable and a very difficult result on the exponential slowness of the evolution of the action variables in weakly perturbed integrable Hamiltonian dynamical systems. As a hypothesis, such estimates were suggested already by J. E. Littlewood (and, possibly, also by G. D. Birkhoff). Today these estimates remain the strongest achievement in the multidimensional Kolmogorov-Arnold-Moser (KAM) theory where Kolmogorov tori do not divide the phase space and where phase trajectories can drift slowly from the gaps between these tori. (In astronomic terms, such a drift can correspond to, for example, the Moon falling on the Earth, or to the disintegration of the solar system; the KAM theory states that such events are unlikely, while the Nekhoroshev theory assures that even if they are going to happen, they will do it extremely slowly, even on a cosmological time scale.) The exponentially small rate of accumulation of perturbations which was proven by Nekhoroshev is the only reason for the existence of planets, asteroids, and comets in the neighbourhood of the so-called chaotic region. In essence, the time by which the parameters of the principal perturbed motion change substantially is estimated to be larger than an exponent of some power of the quantity which is inversely proportional to the perturbation. Nekhoroshev's theory is based on a wonderful combination of ideas from the theory of Diophantine approximations on subvarieties of the Euclidean space and on estimates for Fourier series sums. Nekhoroshev distinguished an important class of unperturbed Hamiltonian systems which he called steep, and this was instrumental to his success. Every analytic subvariety of an Euclidean space which does not belong to any hyperplane turns out to be automatically steep. Nekhoroshev's estimates are given in terms of rational numbers which he called steepness indices and which characterize the curvature of the variety.

This work of N. N. Nekhoroshev was honoured by the Moscow Mathematical Society award in 1974 and the A. N. Kolmogorov prize of the Russian Academy of Sciences (RAS) in 1997. Nominating N. N. Nekhoroshev for the RAS corresponding member election in 1997, the RAS member V. I. Arnold writes: «The obvious drawback of N. N. Nekhoroshev's candidacy is his lacking the [higher] doctor degree [D.Sci]. I explain it by the extremely high standards that he demands of himself. And indeed, the level of his Ph.D thesis surpasses that of the majority of D.Sci theses.» The theory developed by Nekhoroshev rapidly became famous. In 1974, N. N. Nekhoroshev was invited as a lecturer at the International Congress of Mathematicians in Vancouver, Canada. His works are regarded as a cornerstone in the theory of Hamiltonian systems and have been included in textbooks and monographs. They have been further developed by many mathematicians, specialists in mechanics, and physicists. N. N. Nekhoroshev contributed also to extending many results of his theory to partial differential Hamiltonian equations. This is a similarly remarkable achievement.

In the following years, N. N. Nekhoroshev continued to work on the theory of Hamiltonian systems. We formulate here one of his results which has turned out to be of great use in soliton studies. Let a Hamiltonian system have a set of commuting first integrals whose number is less than the number of degrees of freedom. Let the phase space contain a smooth compact connected variety whose dimension equals the number of integrals and which is invariant with respect to all Hamiltonian vector fields generated by these integrals. Then, under certain nondegeneracy conditions, this variety is a torus which belongs to a family of invariant tori parameterized by the constant values of the integrals. In the phase space, all these tori together form a symplectic variety, and the restriction of the system on this variety becomes a standard completely integrable system.

N. N. Nekhoroshev attended many international conferences and worked as an invited professor or researcher in a number of universities abroad. He visited Canada, Cuba, Poland, the United States, Germany, England, and during his last years—France and Italy where he became professor of the Milano university for three years.

One of the important distinctive qualities of N. N. Nekhoroshev was his interest and willingness to work on the development of concrete problems in applied mechanics and physics. So in 2001, during his first encounter with B. I. Zhilinskií at the conference held at the Isaac Newton Institute in Cambridge, UK, he became interested in the mathematical aspects of qualitative models of concrete atomic and molecular quantum systems, such as the hydrogen atom in external homogeneous fields or the system of several coupled angular momenta. His attention was directed primarily to the application of the methods in the theory of integrable Hamiltonian dynamical systems to systems where action-angle variables could not be defined globally.

The problem of the existence of global action-angle variables was analyzed in the first scientific work by N. N. Nekhoroshev which we mentioned above. He had realized already at that time that monodromy was the most basic obstacle to the existence of such variables. Monodromy remained, however, an abstract mathematical concept without possible applications in physical examples which were not given. It was later, thanks to the independent work by J. J. Duistermaat in 1980 and its subsequent development by R. H. Cushman, that physicists became interested in Hamiltonian monodromy and its quantum analogs. Many fundamental systems turned out to be essentially related to monodromy. It should be noted that Duistermaat himself (1942–2010) understood well the contribution by Nekhoroshev, had cited Nekhoroshev in the 1980 paper, and followed with great consideration his late works on monodromy.

During his visit to the French university of Littoral in Dunkirk (Université du Littoral - Côte d'Opale, Dunkerque), where he worked as invited professor in the end of 2001 and the beginning of 2002, N. N. Nekhoroshev became interested in the formal heuristic generalizations of quantum monodromy which were suggested by his physicist colleagues. Such generalizations follow from the study of joint eigenspectra of two quantum operators, energy and momentum, which correspond to the first integrals of the underlying classical system. In the space of all possible energy-momentum values, i.e., in the base of the integrable fibration, the spectrum forms a two-lattice. Monodromy manifests itself as a (point) *defect* of this lattice centered at the image of the strongly singular point. To uncover monodromy, one considers a closed path in the regular domain of the lattice which goes around the image of this point (the nontrivial cycle of the fundamental group of the base of the fibration). Subsequently, it became customary to demonstrate the defects using a family of elementary (minimal) cells which deform continuously along such a path. Note that the choice of the original cell corresponds unambiguously to the choice of the basis in the first homology group of regular fibres. Monodromy relates the original and the final cells.

In the beginning of 2002, after considering possible lattices of systems with resonance 1:2, the idea of a «fractional» defect was put forward. To investigate if and how this can occur, an appropriate dynamical system, the $1{:}(-2)$ resonance oscillator with high order compactifying term was suggested jointly. At first, N. N. Nekhoroshev himself was categorically objecting to the idea that monodromy could be «fractional». (And indeed, we deal with automorphisms of the first homology group H_1 of the torus which are given by matrices over integers!) A concrete calculation for the quantum analog of the $1{:}(-2)$ system helped him to understand what was happening and to interpret it in terms of the subgroup

H_1/Z_2. Subsequently, N. N. Nekhoroshev came to formulate his idea of the proof of usual and fractional monodromy on concrete examples. His approach gives a universal description which relies on the geometric properties of the fibration. The dynamics is used in his proof only to orient the cycles in the homology group (to define the so-called «sign» of monodromy). Essentially, N. N. Nekhoroshev gave a geometric proof of Hamiltonian monodromy which can be related directly to the monodromy concepts in other domains of mathematics. Note that while «usual» Hamiltonian monodromy in the simplest nondegenerate case is locally analogous to the Picard-Lefschetz monodromy of the A_1 singularity, there is no analog of fractional monodromy in the singularity theory. So, analyzing concrete dynamical systems, N. N. Nekhoroshev generalized the very concept of monodromy. His definition allows for using complete subgroups of the first homology group of regular fibres.

N. N. Nekhoroshev understood well the necessity of solid complete proofs, but most of all, he aimed at conceptual cleanness and universality. He used to say: «Let us make sure that our formulation is right! As for the (elegant) proofs—others will follow and come up with them». And they did, fortunately. Afterwards, it took a number of years and a series of joint meetings and discussions in order to complete the proofs for the concrete examples. Very sharp and attentive to details, Nekhoroshev reached at the same time far ahead and widely to other domains that he understood and felt profoundly. Aiming at ultimate clarity, he could iterate painstakingly over and over the same paragraph, lemma, or definition. Sometimes, the result of the whole day spent together amounted to a half-page, and sometimes it was—rejected the next day.

N. N. Nekhoroshev became firmly convinced of the originality of the concept of fractional (generalized) monodromy and began to work actively on its rigorous mathematical foundations. It was the subject of his large last manuscript which, sadly, had not come to light during his life. This year, his work is published unabridged in Russian in the journal Russ. J. Nonlin. Dyn. [vol. 12(3), pp. 385–513 (2016)]. Responding to the referee in early April 2008, Nekhoroshev characterized his work: «I believe that the paper will be useful not only because of its main result, but also because it illustrates a number of rather powerful and universal tools to compute monodromy using a special kind of global section which is similar to the Poincaré section for the trajectories of vector fields.» At the same time, the idea of fractional monodromy has become recognized and, as Nikolaí Nikolaevich has anticipated, new proofs and interpretations followed. We can say without exaggerating that generalized monodromy was one of the N. N. Nekhoroshev's principal scientific interests during the last years of his life. He is a key figure in the development of the mathematical theory of this phenomenon. The untimely passing away of N. N. Nekhoroshev on October 18, 2008 has left many possible directions of further generalizations and applications without continuation. He outlined these possibilities and discussed them with his colleagues. Full of new ideas, he intended to develop them during his new visit to the university of Littoral in May 2008. One week before leaving, a rapid and unexpected degradation of his health made him cancel the trip. The far reaching plans remained unaccomplished.

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