Lecture 12:
Fraunhofer diffraction by a single slit

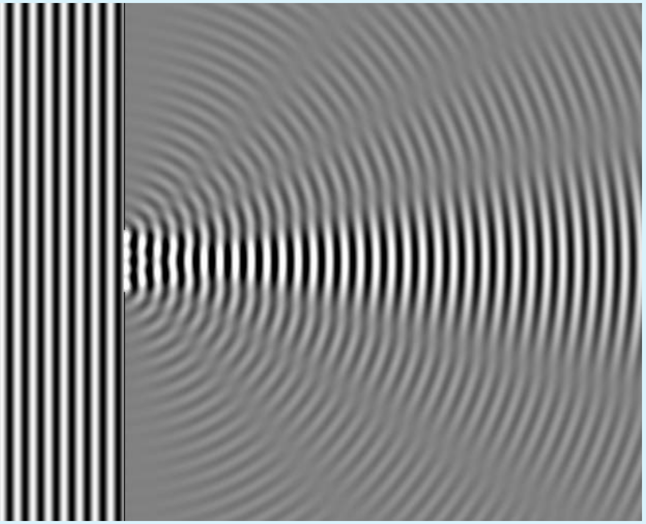
Lecture aims to explain:

1. Diffraction problem basics (reminder)

2. Calculation of the diffraction integral for a long slit

3. Diffraction pattern produced by a single slit

4. Use of a convex lens for observation of Fraunhofer diffraction pattern



Diffraction problem basics (reminder)

Diffraction basics

Problem: A propagating wave encounters an obstacle (i.e. a distortion of the wave-front occurs). How will the distortion influence the propagation of the wave?

Fraunhofer diffraction: the resultant wave is measured very far away from the place where the wave-front was distorted
(R>>size of the obstacle)

General approach:
(i) Split the wave-front into infinitely small segments and consider emission of secondary wavelets (using **Huygens principle**)

(ii) Fix the direction of observation and calculate the combined **electric field** of **all** wavelets from **all** original segments taking into account **difference in optical path length** and **amplitude**

Why do we study diffraction on slits, circular apertures etc: to understand basics, and due to high relevance to applications

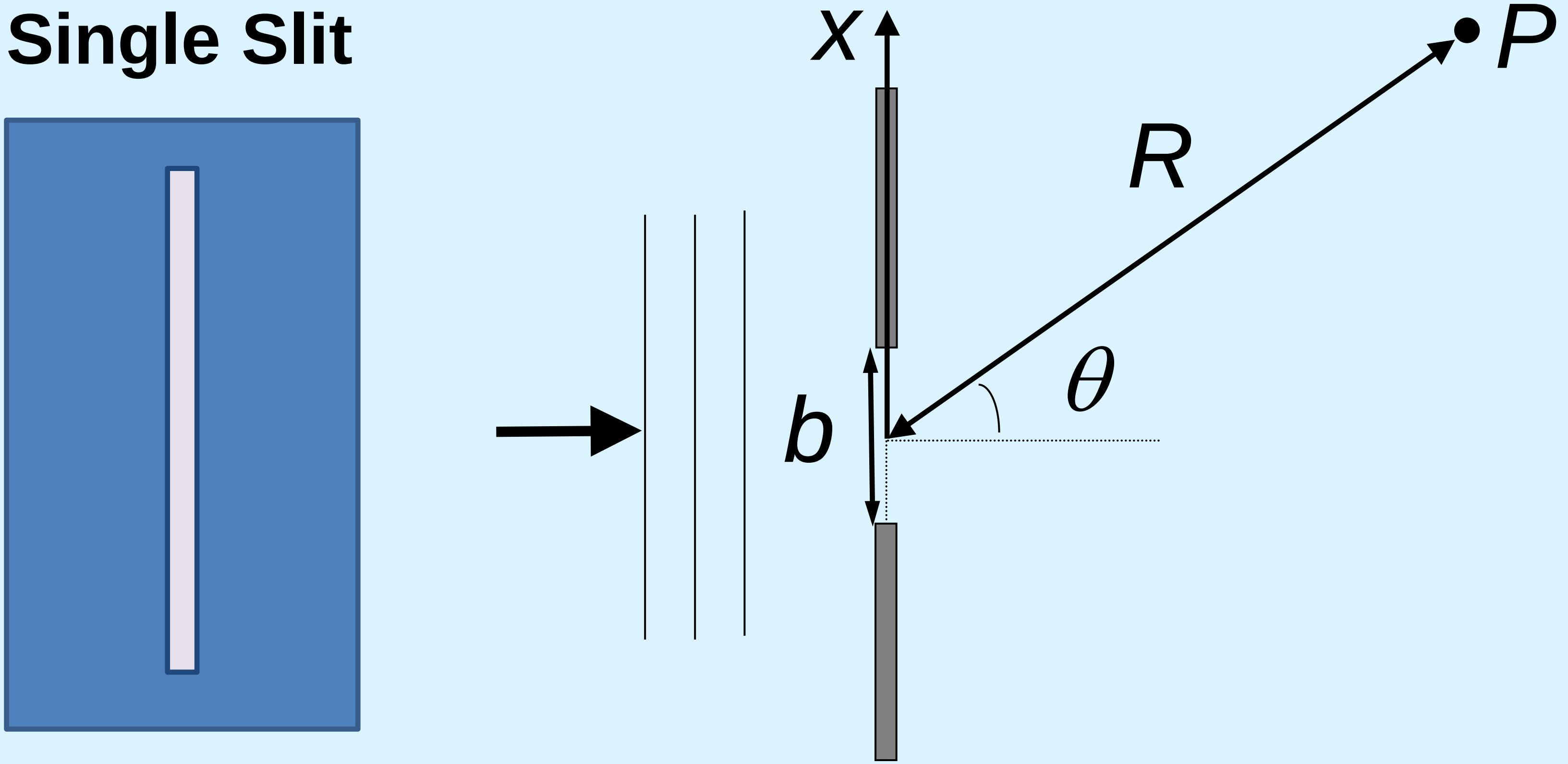
History of discovery of diffraction

The effects of diffraction of light were first observed and characterized by Francesco Maria Grimaldi in the 17th century. James Gregory (1638–1675) observed the diffraction patterns caused by a bird feather. Thomas Young performed a celebrated experiment in 1803 demonstrating interference from two closely spaced slits. Augustin-Jean Fresnel did systematic studies and calculations of diffraction around 1815. This gave great support to the wave theory of light that had been developed by Christiaan Huygens in the 17th century.

Joseph von Fraunhofer was a famous German optician, who perfected manufacture of highest quality glass in Bavaria.

Calculation of the diffraction integral for a long slit

The Single Slit



Electric field measured at the **distant** point *P*

$$E(\theta) = \frac{\mathcal{E}_L}{R} \int_{-b/2}^{+b/2} \sin[\omega t - k(R - x\sin\theta)]dx$$

$\mathcal{E}_L$ "source" strength per unit length

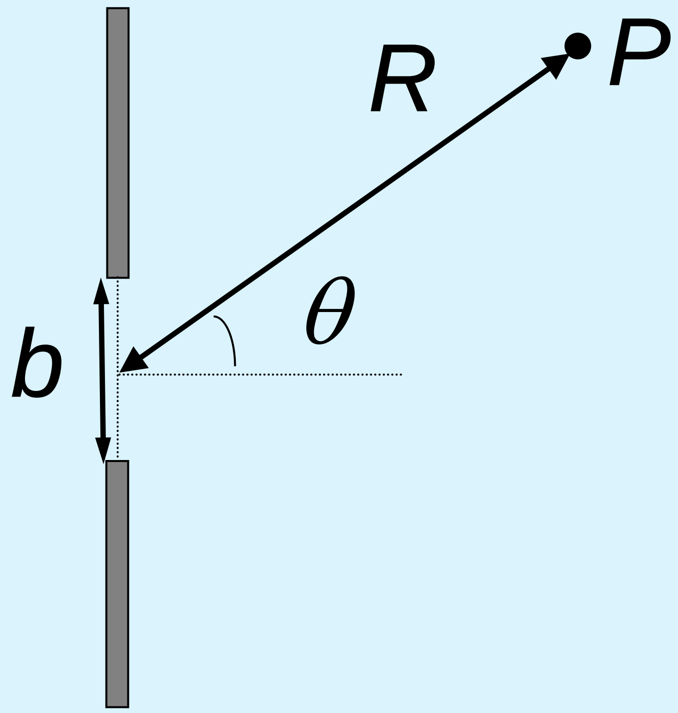
The irradiance produced by the diffracted wave

Light diffracted by a long slit of width *b* produces **irradiance** at a distant position *P* in the direction with an angle θ:

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

where

$$\beta = (kb/2)\sin\theta$$



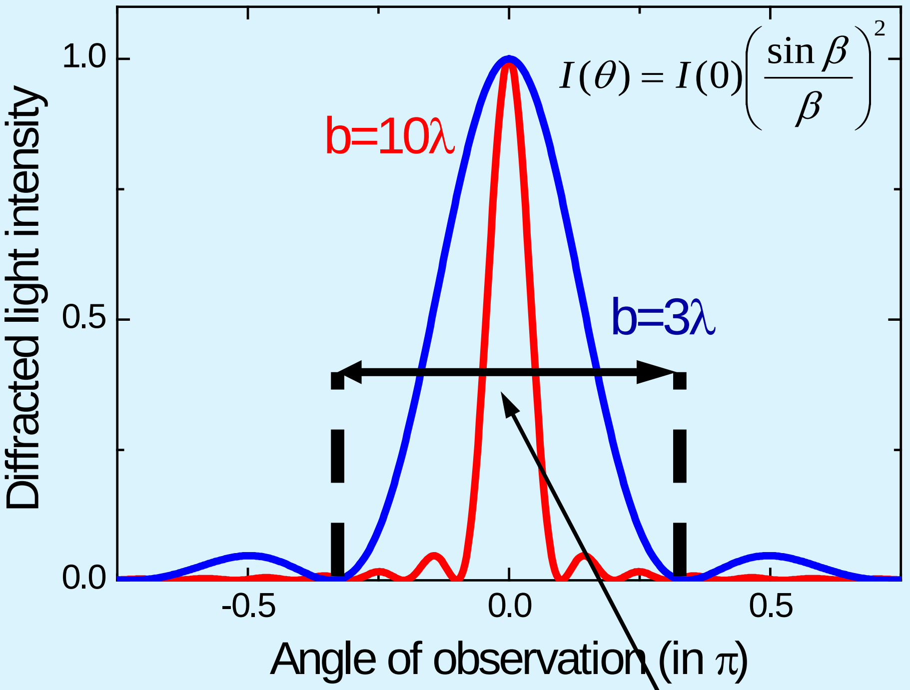
Diffraction pattern produced by a long slit

Diffraction pattern

Central maximum: in the direction of original light propagation at *θ=0*

Zeros: $\beta = \pm\pi, \pm 2\pi, \pm 3\pi$ … i.e. for *m≠0:*

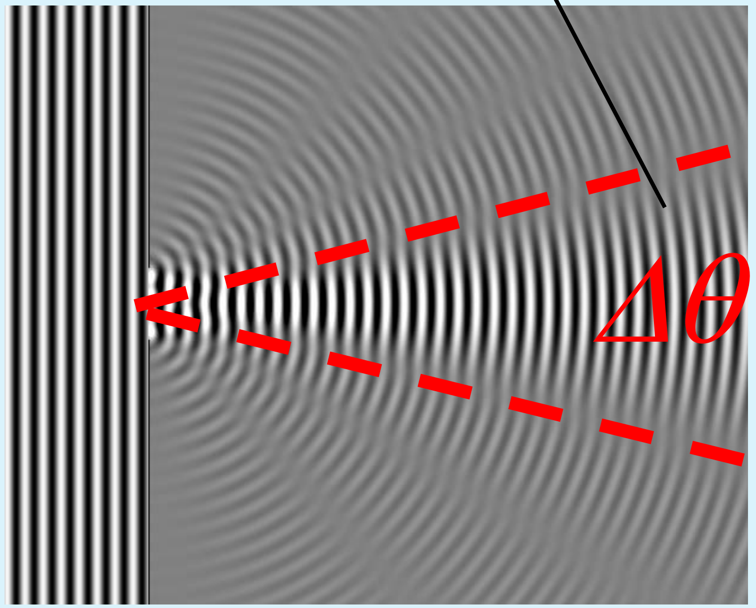
$$\beta = \frac{\pi b}{\lambda}\sin\theta = m\pi$$



The angular width of the central maximum is defined by:

$$\sin\theta_{\pm 1} = \pm\frac{\lambda}{b}$$

For *b*>>*λ* $$\Delta\theta = 2\frac{\lambda}{b}$$

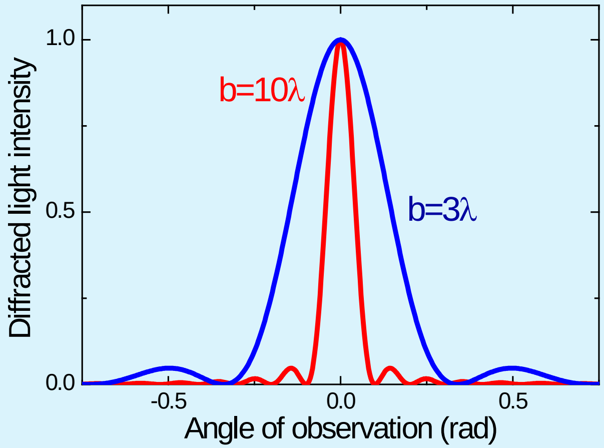


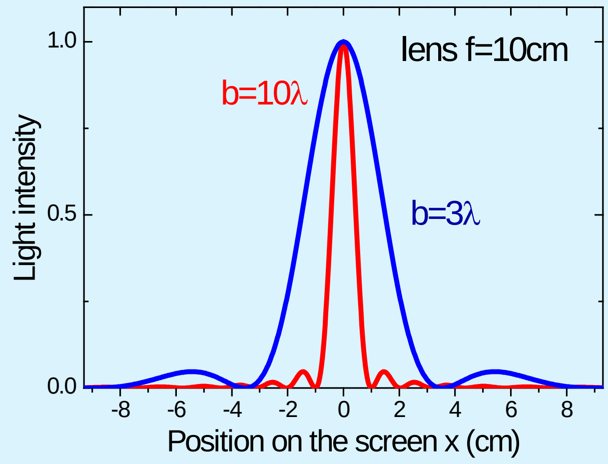
http://surendranath.tripod.com/Applets/Optics/Slits/SingleSlit/SS.html

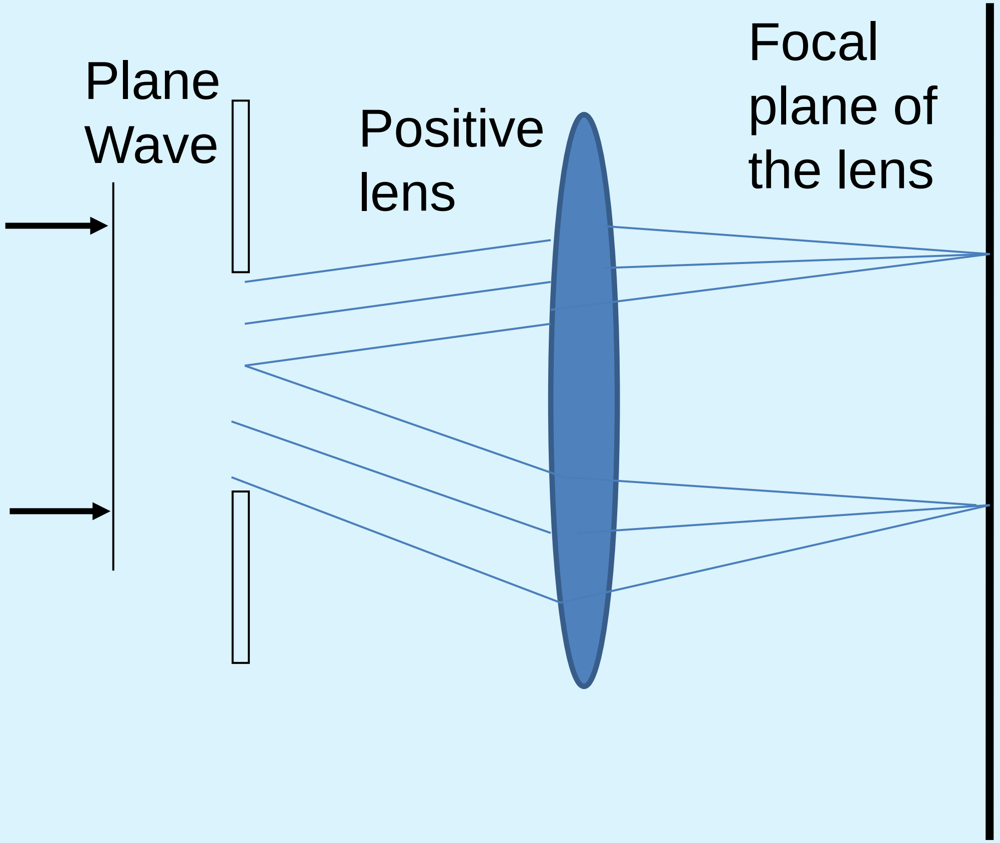
Use of a convex lens for observation of Fraunhofer diffraction pattern

Observation of Fraunhofer diffraction

The angular dependence of the diffracted light intensity is replaced by the function of spatial coordinates in the focal plane of the positive lens.
Position on the screen (f- focal length of the lens) $x = f \tan\theta$







Example 12.1

Light is incident on a screen with a 0.1 mm wide slit. The diffraction pattern is obtained in the focal plane of a lens positioned a few cm behind the screen. The focal length of the lens is 10 cm. Find the width of the central maximum in the intensity of the diffraction pattern for (i) blue and (ii) red light.

To see how diffraction on a slit works visit:
http://surendranath.tripod.com/Applets/Optics/Slits/SingleSlit/SS.html

SUMMARY

Electric field measured at a distant the point for a single slit

$$E(\theta) = \frac{\varepsilon_L}{R} \int_{-b/2}^{+b/2} \sin[\omega t - k(R - x\sin\theta)]dx$$

ε_L "source" strength per unit length

Light diffracted by a long slit of width *b* produces **irradiance** at a distant position *P* in the direction with an angle θ:

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2 \quad \text{where} \quad \beta = (kb/2)\sin\theta$$

The angular width of the central maximum is defined by:

$$\sin\theta_{\pm 1} = \pm\frac{\lambda}{b}$$

For *b*>>λ

$$\Delta\theta = 2\frac{\lambda}{b}$$

The angular dependence of the diffracted light intensity is replaced by the function of spatial coordinates in the focal plane of the positive lens. Position on the screen (f- focal length of the lens)

$$x = f\tan\theta$$