NEUTRAL HYDROGEN-LIKE SYSTEM IN A MAGNETIC FIELD

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Explicit formulae are derived relating the energies and the ionisation potentials of an arbitrary neutral two-particle sys- [redacted]

It has been shown [1–3] that the Schrödinger equation for a neutral two-particle system in a uniform magnetic field $\boldsymbol{B}$ may be reduced to a one-particle equation describing the relative motion. This equation depends on the vector $\boldsymbol{P}$, which is the projection of the total momentum vector on the plane orthogonal to the field direction, and has the form:

$$\left[\frac{P^2}{2M}-\frac{\hbar^2}{2\mu}\Delta_r+\frac{q}{cM}\boldsymbol{P}\times\boldsymbol{B}\cdot\boldsymbol{r}-i\hbar\frac{q\beta}{2c\mu}\boldsymbol{B}\cdot\boldsymbol{r}\times\nabla_r+\frac{q^2}{8c^2\mu}(\boldsymbol{B}\times\boldsymbol{r})^2-\frac{q^2}{|\boldsymbol{r}|}\right]\psi(\boldsymbol{r})=\mathcal{E}(\boldsymbol{B})\psi(\boldsymbol{r})\,,$$

$$M=m_++m_-\,,\quad \mu=m_+m_-/M\,,\quad \beta=(m_+-m_-)/M\,,\quad q=q_+\,. \tag{1}$$

Here $\boldsymbol{r}$ is the vector directed from the positive particle to the negative one. All quantities related to the negative and positive particles are labeled correspondingly by (–) and (+). For the hydrogen atom m_- is the electron mass m_e and m_+ is the proton mass m_p.

Recently a particular form of eq. (1) for the case $\boldsymbol{P}=0$ was derived [4]. It was pointed out that eq. (1) coincides with the Schrödinger equation for a hydrogen atom of infinite nuclear mass if one replaces μ by m_e and puts $\beta = 1$. This observation was used to point out that the energy corrections due to the finiteness of the proton mass are of the order of m_e/m_p, i.e. they are significantly smaller than the earlier incorrect numerical values given in ref. [5].

In the present note we give explicit formulae, which enable one to calculate the energies and the ionisation potentials of an arbitrary neutral two-particle system in a magnetic field $\boldsymbol{B}$ using the corresponding quantities for a hydrogen atom of infinite nuclear mass in a magnetic field $(m_e/\mu)^2B$, where $B=|\boldsymbol{B}|$. We consider the case $\boldsymbol{P}=0$. So we take into account only the finiteness of the particle masses. The more complicated case $\boldsymbol{P}\neq 0$ was discussed in refs. [1–3] but still requires some further clarification.

To derive these formulae we put $\boldsymbol{P}=0$ and transform eq. (1) to cylindrical coordinates ρ, φ, z with the z axis directed along the magnetic field. The solution of eq. (1) in these coordinates has the form $\psi(\rho, z, \varphi) = \phi_m(\rho, z)\exp(im\varphi)$, where the functions $\phi_m(\rho, z)$ describe the states with a definite projection of the angular momentum vector on the field direction and satisfy

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{m^2}{\rho^2}+\frac{\partial^2}{\partial z^2}\right)+\frac{\hbar q\beta}{2c\mu}Bm+\frac{q^2}{8c^2\mu}B^2\rho^2-\frac{q^2}{(\rho^2+z^2)^{1/2}}\right]\phi_m(\rho,z)=\mathcal{E}_m(B)\phi_m(\rho,z)\,. \tag{2}$$

Let us now use the scale transformation $\rho_1=\lambda\rho$, $z_1=\lambda z$ with $\lambda=\mu/m_e$ to transform eq. (2) into

$$\left[-\frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial\rho_1^2}+\frac{1}{\rho_1}\frac{\partial}{\partial\rho_1}-\frac{m^2}{\rho_1^2}+\frac{\partial^2}{\partial z_1^2}\right)+\frac{\hbar q}{2cm_e}\left(\frac{m_e^2}{\mu^2}B\right)m+\frac{q^2}{8c^2m_e}\left(\frac{m_e^2}{\mu^2}B\right)^2\rho_1^2-\frac{q^2}{(\rho_1^2+z_1^2)^{1/2}}\right]\phi_m(\rho_1,z_1)$$
$$=\left[\frac{m_e}{\mu}\mathcal{E}_m(B)+(1-\beta)\frac{\hbar q}{2cm_e}\left(\frac{m_e^2}{\mu^2}B\right)m\right]\phi_m(\rho_1,z_1)\ . \tag{3}$$

From comparison of eq. (3) with the Schrödinger equation for a hydrogen atom of infinite nuclear mass in a magnetic field B_1,

$$\left[-\frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}-\frac{m^2}{\rho^2}+\frac{\partial^2}{\partial z^2}\right)+\frac{\hbar q}{2cm_e}B_1m+\frac{q^2}{8c^2m_e}B_1^2\rho^2-\frac{q^2}{(\rho^2+z^2)^{1/2}}\right]\phi_m(\rho,z)=E_m(B_1)\phi_m(\rho,z)\ , \tag{4}$$

one can easily see that the right-hand side of eq. (3) contains the eigenvalue of eq. (4) for $B_1=(m_e/\mu)^2B$. So we obtain the desired formula

$$\mathcal{E}_m(B)=\frac{\mu}{m_e}E_m\left(\frac{m_e^2}{\mu^2}B\right)-\frac{\hbar q}{cm_+}Bm\ . \tag{5}$$

In a similar way one can obtain the relation for the ionisation potentials:

$$\mathcal{E}_m^{\mathrm{I}}(B)=\frac{\mu}{m_e}E_m^{\mathrm{I}}\left(\frac{m_e^2}{\mu^2}B\right)+\frac{\hbar q}{cm_+}Bm\ , \tag{6}$$

where $\mathcal{E}_m^{\mathrm{I}}$ and E_m^{I} are the ionisation potentials of the neutral two-particle system and of the hydrogen atom of infinite nuclear mass, respectively.

It must be noted in conclusion that the accurate consideration of the finite (nucleus) mass corrections to the energy is important for such systems as positronium, muonium, etc., which may exist near pulsars. The obtained formulae enable one to study such exotic systems using the well-known numerical results for the energy levels of a hydrogen atom of infinite nuclear mass.

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